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Don't just stand there—teach Fermi problems!

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Abstract

Fermi problems, or order of magnitude estimates, are often used in introductory physics courses. In this paper I will show that first year students studying physics at university do not arrive with the skill set to solve these problems, and they have to be actively taught how to solve them. Once they have been shown how to solve Fermi problems, the above-average students in the class usually perform slightly better on Fermi problems in examinations than they do on more traditional physics problems. However, many students who are below average in overall grade find the Fermi problems even more challenging than the traditional problems.

Introduction

Most students taking introductory physics courses in the 21st century have always had access to calculators and computers, and so are not familiar with having to make an order of magnitude estimate. This type of problem is also known as a back-of-envelope calculation, or a Fermi problem, due to Enrico Fermi's aptitude for posing and solving such calculations [1, 2]. The reason for the historical emphasis on the order of magnitude calculation as an essential skill in physics [3] was the lack of computing power available to solve complex problems; an order of magnitude calculation could confirm the feasibility of a model and determine whether an onerous and time-consuming full calculation was required. In the last few decades, it has become easy to do a full calculation without the preliminary order of magnitude estimate, and so the skill set required to make an estimate has tended to lapse. Students now expect to be able to solve a problem exactly, often to an unrealistic level of precision, because they have ready access to calculators, computers and mathematical software tools.

In order to solve a Fermi problem, one has to synthesize a physical model, examine the physical principles which are in operation, determine other constraints such as boundary conditions, decide how simple the model can be while still maintaining some realism, and only then apply some rough estimation to the problem. This type of exercise is exactly the skill set which professional physicists develop during their training, and it has been shown that this skill is often not fully developed until sometime during graduate studies [4]. Clearly, the sooner we can instil a professional methodology into our students the better [5].

Inexperienced students often solve problems using the 'hunt for an equation containing known and unknown variables' strategy. This is usually successful at an introductory level, but reduces the physics problem to merely substituting numbers into an equation, an exercise in mathematics and

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arithmetic. The actual 'doing the physics' part of looking for the physical model and physical principles relevant to the problem is missing. This strategy also reinforces the perception of students that physics is synonymous with mathematics. Of course it is easy to structure teaching material to allow this method to flourish. If an assignment is set with questions from a particular chapter in the textbook, the canny student will realize that there are only a limited number of equations in that chapter, and so can easily scan for the appropriate equation which fits the given data and required unknown. I suspect most teachers do this, and I must confess to being guilty of this practice myself.

In recent years I have taught the Fermi problem solving method to my calculus-based introductory physics course, which is a 13 week class, with 36 lectures of 50 min each, weekly assignments and a laboratory-based experimental component. The students in this class are predominantly in the common engineering first year at the University of Saskatchewan, and take physics as an elective. From discussions with students over the years, I can confirm that the 'hunt for the equation' strategy is extremely common, and has been rewarded by the high school system, as many of the students enter university with excellent grades.

Teaching methodology

My method of teaching is to pose an in-class Fermi problem to the students, allow them 10 min to formulate an answer, conduct a poll of the range of answers, and finish with an explanation of my own answer and a discussion of the results. Usually the whole process takes about 25 min. I allow the students to work in small groups, giving an element of peer instruction, and I circulate around the class, giving advice and discussing the problem with the students. This seems to work well with a class size of up to 80 students. I do not allow the use of calculators or computers during the exercise, which usually causes some consternation [6]. In addition to the in-class exercises, a Fermi problem is set on most of the weekly assignments.

These classes are taught in the mid-afternoon, when the students have already sat through several lectures. I find that a traditional lecture is unsatisfactory, as the class has a limited attention span. The in-class exercise helps to change the pace of the class and forces the students to take an active role. This format also promotes student interaction with the teacher, so that by the end of the course, the students are asking many questions in class, and not just during the in-class exercises. This significantly enhances the teaching and learning environment, and makes the class much more stimulating for both the students and their teacher! I also found that the in-class exercise quickly exposes gaps in the knowledge of the student and what the teacher assumes that they already know. In my first ever in-class exercise, I found out that 90% of the class did not know the equation for the area of a circle of radius r, something I had assumed that they would have known from high school.

Results and discussion

The mid-term examination is early in the course, when the students are relatively inexperienced at solving Fermi problems. They have usually done two of the in-class exercises and two or three assignment problems. The test is 2 h long, with five problems being set, one of which is a Fermi problem. The other questions are typical of those found in the course textbook [7]. I will refer to them as 'traditional' physics problems. In order to gauge the relative performance of students on Fermi questions, I define a relative performance factor R for each student:

$$R = \frac{x - \bar{x}_{\mathrm{nf}}}{\bar{x}_{\mathrm{nf}}}, \qquad (1)$$

where x is the score on the Fermi problem, and $\bar{x}_{\mathrm{nf}}$ is the mean score on the non-Fermi problems on the paper. All problems are marked out of 10. A positive R-factor means that the student performs better on Fermi problems than on the more traditional problems. R-factors for each of the 116 students who sat both the mid-term examination and the final examination were calculated. I did not calculate R-factors for the Fermi problems set in assignments, as these are often solved collaboratively, whereas the examination answers are a measure of individual aptitude. The marking scheme for the Fermi problem emphasizes the importance of constructing a well reasoned, logical argument at each step of the calculation, and not on the value of the final answer.

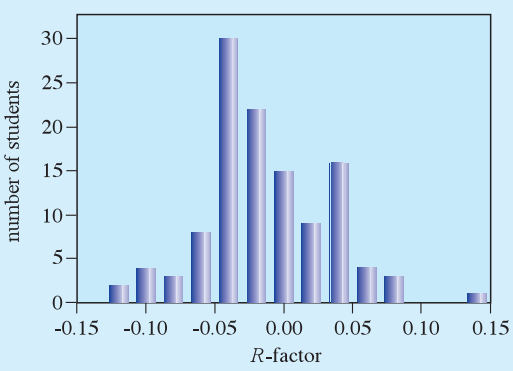


Figure 1. Distribution of R-factors for the mid-term examination.

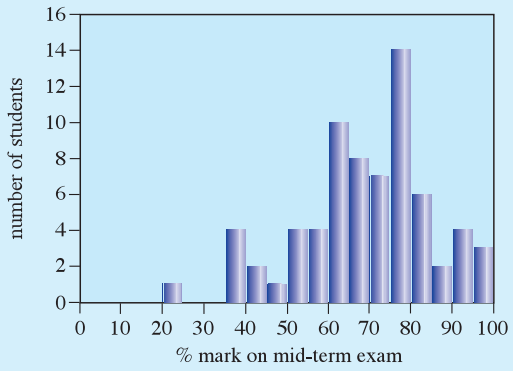


Figure 2. Distribution of marks for students with a negative R-factor in the mid-term examination.

In figure 1, I show the R-factors calculated from the mid-term examination. We can see that there is a broad distribution of R-factors, with the mean R-factor being -0.02 and approximately equal proportions of students above and below this mean value. This means that, on average, the students underperformed on the Fermi question (see appendix A) compared to their performance on the other questions.

A closer analysis of students who scored poorly on the Fermi questions (a negative R-factor) reveals that these students come from the whole ability range, if their performance is assessed on the more traditional problems. Figure 2 shows that a sizeable number of students who score well overall are included in the group with a negative R-factor. This suggests that some students across the entire ability range had some trouble adapting to the new method of problem solving.

After several more weeks of work, with four or five more in-class exercises and several weekly assignment problems, the students sat the final examination. A single Fermi question (see appendix B) out of a total of nine set questions was posed on this 3 h test. Figure 3 shows the R-factor distribution for the final examination. The distribution is now strikingly bimodal, with a normal distribution based around an average R-factor of 0.03, and a second distribution clustered around an average R-factor of -0.04.

The bimodality suggests that the majority of students have now had sufficient practice at solving Fermi problems to perform better than

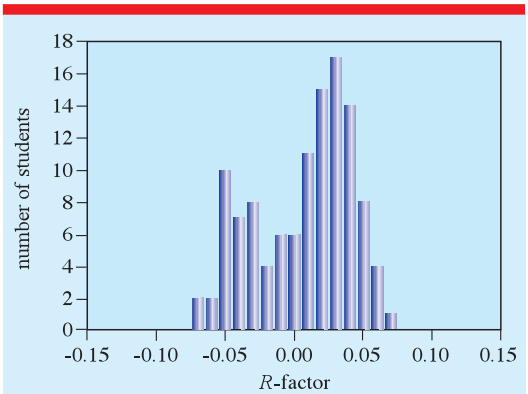


Figure 3. Distribution of R-factors from the final examination.

they do on the rest of the examination questions. However, a minority of students, in the negative cluster, are still having difficulties and have not adapted to a new problem solving strategy.

Figure 4 shows the marks distribution for the group with negative R-factors on the final examination. Their average mark is clustered around the 50% level, a marginal pass.

If we compare this with the marks distribution of the group of students with a positive R-factor on the final examination, in figure 5, we can see that this group has a much higher average mark of 69%, and that very few of this group score less than 50%. The proportion of students in this group is two thirds of the total, compared with half with a positive R-factor score on the mid-term examination. Some students have improved their performance and are now able to solve Fermi

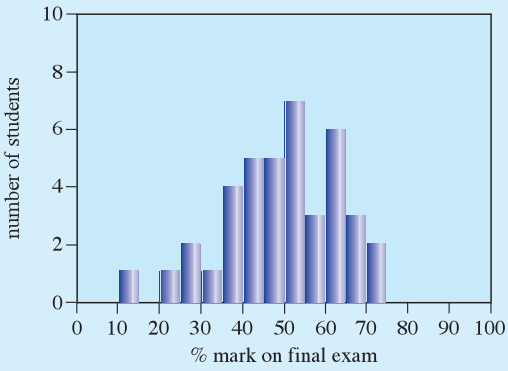


Figure 4. Distribution of marks for students with negative *R*-factors on the final examination.

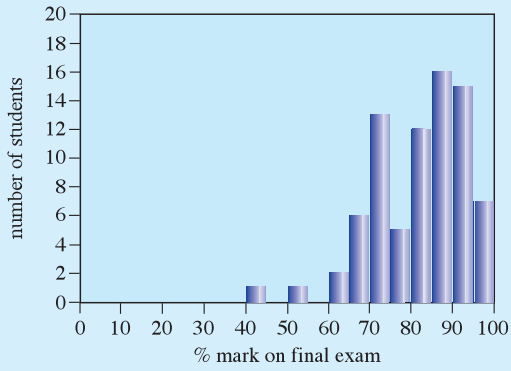


Figure 5. Marks distribution for students with a positive *R*-factor on the final examination.

problems better than they did on the mid-term examination.

Conclusions

When analysing the results of this study, we must, of course, note that this was a snapshot of a single year group of 116 students, and the results are from a single Fermi problem set on each of the two examinations. One should be cautious about over-interpreting the results. Nevertheless, some conclusions can be drawn.

The skill set needed to solve a Fermi problem is not a skill which most students have acquired while at high school; it is a skill which needs to be taught. Many of the better students score badly on Fermi problems until they have sufficient experience. Since the problem solving skills are precisely those required for students taking advanced physics, it is important to introduce them into the course of study as early as possible, preferably at the high school level. Once students have had sufficient experience in solving problems for themselves and seeing how the teacher solves them, the above-average students are able to adopt this problem solving strategy and achieve marks at least as high as on the traditional physics problems. However, a group of students, who are below average on traditional problems, score even more poorly on Fermi problems, as they have not managed to adapt their problem solving skills to the new approach. A colleague who teaches a similar problem solving course in physical chemistry to the same group of students has also reported informally that he finds this in his class too [8]. This type of problem solving skill is not an innate one: it requires nurturing and developing. As teachers, we need to ensure that our students receive the appropriate training in solving physics problems. In an ideal world, students would learn this while at school. To teach Fermi problems, I use several different techniques, worked examples in class, practice problems and feedback in class, and practice problems and feedback in weekly written assignments. The active method of teaching in the classroom provides a vehicle for improving the interaction between teacher and students and provides a more stimulating environment for both teacher and students.

On the basis of these results, I have adjusted the course content and delivery, so that students have more experience in solving Fermi problems in class and on assignments before they have to answer questions in an examination. I also plan to use a classroom response system during 2008, so that I can compile statistics on the range of answers obtained in the classroom exercises.

Answering a Fermi problem, with a well argued persuasive case, is a classic case of rhetoric. It might be interesting to see if students from disciplines other than physics (such as law) which put an emphasis on such training would perform better than physics students when faced with a Fermi problem which does not require detailed knowledge of physics.

Acknowledgments

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Appendix A

Estimate to the nearest order of magnitude the kinetic energy lost in a head-on collision between a car and a truck on the highway. You may assume that both vehicles are stationary after the collision. The legal speed limit on the highway is 110 km h^{-1}. Your answer must be expressed in joules. State clearly in your answer any assumptions which you use. Where does this energy go after the collision?

Appendix B

The power output of the sun is 3.92×10^{26} W.

(a) How much mass does the sun lose per second?
(b) Estimate the change in radius of the sun after 5×10^9 years, assuming a constant power output.

Useful information: current radius of sun = 695 000 km, current mass of sun = 1.99×10^{30} kg.

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References

[1] Lan B L 2002 *Eur. J. Phys.* **23** L29–31
[2] Efthimiou C J and Llewellyn R A 2007 *Phys. Educ.* **42** 253–61
[3] Reif F and St. John M 1979 *Am. J. Phys.* **47** 950–6
[4] Van Heuvelen A 1991 *Am. J. Phys.* **59** 891–8
[5] Hestenes D 1987 *Am. J. Phys.* **55** 440–54
[6] For a none-too-serious commentary on the class see: Robinson A W 2006 Back to the envelope *Phys. World* **19** (11)
[7] Serway R A and Jewett J W 2002 *Physics for Scientists and Engineers with Modern Physics* 6th edn (Philadelphia, PA: Thomson Scientific) ISBN 0-534-40844-3
[8] Reid R S 2007 private communication



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